CHAPTER 7

Gravitation

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

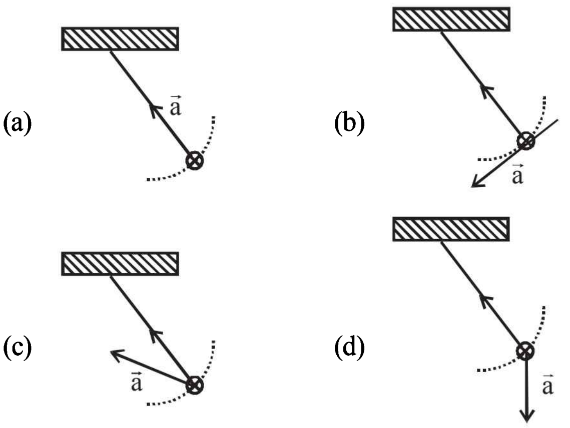
1. The numerical value of the angular velocity of rotation of the earth should be ....rad/s in order to make the effective acceleration due to gravity equal to zero. *(1984 - 2 Marks)*
2. A geostationary satellite is orbiting the earth at a height of 6 R above the surface of the earth, where R is the radius of the earth. The time period of another satellite at a height of 2.5 R from the surface of the earth is ....hours. *(1987 - 2 Marks)*
3. The masses and radii of the Earth and the Moon are M_1, R_1 and M_2, R_2 respectively. Their centres are at a distance d apart. The minimum speed with which a particle of mass m should be projected from a point midway between the two centres so as to escape to infinity is .... *(1988 - 2 Marks)*
4. A particle is projected vertically upwards from the surface of earth (radius R_e) with a kinetic energy equal to half of the minimum value needed for it to escape. The height to which it rises above the surface of earth is.... *(1997 - 2 Marks)*

B True/False

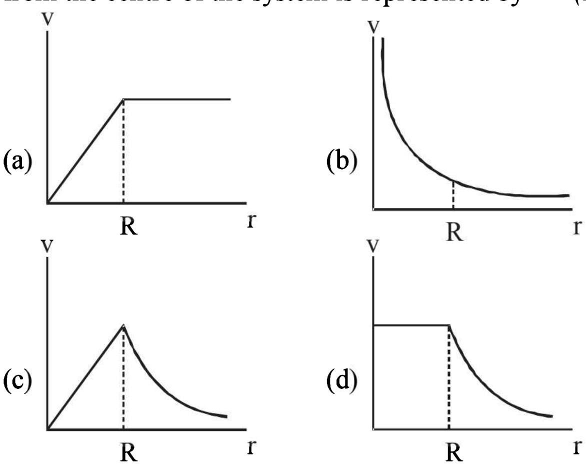
1. It is possible to put an artificial satellite into orbit in such a way that it will always remain directly over New Delhi. *(1984 - 2 Marks)*

C MCQs with One Correct Answer

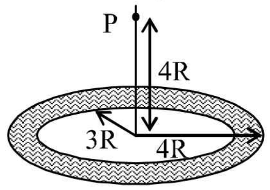
1. If the radius of the earth were to shrink by one percent, its mass remaining the same, the acceleration due to gravity on the earth's surface would *(1981 - 2 Marks)*
   (a) decrease (b) remain unchanged
   (c) increase (d) be zero
2. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth, is *(1983 - 1 Mark)*
   (a) $\frac{1}{2}mgR$ (b) $2mgR$ (c) mgR (d) $\frac{1}{4}mgR$
3. If the distance between the earth and the sun were half its present value, the number of days in a year would have been *(1996 - 2 Marks)*
   (a) 64.5 (b) 129 (c) 182.5 (d) 730
4. A geo-stationary satellite orbits around the earth in a circular orbit of radius 36,000km. Then, the time period of a spy satellite orbiting a few hundred km above the earth's surface (R_{earth} = 6,400km) will approximately be *(2002S)*
   (a) 1/2 hr (b) 1 hr (c) 2 hr (d) 4 hr
5. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector $\vec{a}$ is correctly shown in : *(2002S)*



6. A binary star system consists of two stars A and B which have time period T_A and T_B, radius R_A and R_B and mass M_A and M_B. Then *(2006 - 3M, –1)*
   (a) if $T_A > T_B$ then $R_A > R_B$ (b) if $T_A > T_B$ then $M_A > M_B$
   (c) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{R_A}{R_B}\right)^3$ (d) $T_A = T_B$
7. A spherically symmetric gravitational system of particles has a mass density $\rho = \begin{cases} \rho_0 & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$ where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as a function of distance r $(0 < r < \infty)$ from the centre of the system is represented by – *(2008)*



8. A thin uniform annular disc (see figure) of mass M has outer radius $4R$ and inner radius $3R$. The work required to take a unit mass from point P on its axis to infinity is *(2010)*



(a) $\frac{2GM}{7R}(4\sqrt{2}-5)$

(b) $-\frac{2GM}{7R}(4\sqrt{2}-5)$

(c) $\frac{GM}{4R}$ (d) $\frac{2GM}{5R}(\sqrt{2}-1)$

9. A satellite is moving with a constant speed 'V' in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is *(2011)*

(a) $\frac{1}{2}mV^2$ (b) mV^2 (c) $\frac{3}{2}mV^2$ (d) $2mV^2$

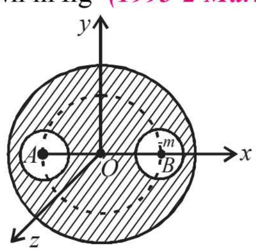
10. A planet of radius $R = \frac{1}{10} \times (\text{radius of Earth})$ has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and a linear mass density 10^{-3} kg m^{-1} into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius of Earth = 6×10^6 m and the acceleration due to gravity on Earth is 10 ms^{-2}) *(JEE Adv. 2014)*

(a) 96 N (b) 108 N (c) 120 N (d) 150 N

D MCQs with One or More than One Correct

1. Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$ *(1989 - 2 Mark)*
   (a) T^2 is proportional to R^3
   (b) T^2 is proportional to $R^{7/2}$
   (c) T^2 is proportional to $R^{3/2}$
   (d) T^2 is proportional to $R^{3/73}$

2. A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit, with their centres at A (–2, 0 ,0) and B (2, 0, 0) respectively, are taken out of the solid leaving behind spherical cavities as shown in fig *(1993-2 Marks)*
   Then :



   (a) The gravitational force due to this object at the origin is zero.
   (b) the gravitational force at the point B (2, 0 ,0) is zero.
   (c) the gravitational potential is the same at all points of circle $y^2 + z^2 = 36$.
   (d) the gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$.

3. The magnitudes of the gravitational field at distance r_1 and r_2 from the centre of a uniform sphere of radius R and mass m are F_1 and F_2 respectively. Then: *(1994 - 2 Marks)*

   (a) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$

   (b) $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$ if $r_1 > R$ and $r_2 > R$

   (c) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 > R$ and $r_2 > R$

   (d) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ if $r_1 < R$ and $r_2 < R$

4. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth. *(1998S - 2 Marks)*
   (a) The acceleration of S is always directed towards the centre of the earth.
   (b) The angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant.
   (c) The total mechanical energy of S varies periodically with time.
   (d) The linear momentum of S remains constant in magnitude.

5. Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q and surface areas A and 4A respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R are V_P, V_Q and V_R, respectively. Then *(2012)*

   (a) $V_Q > V_R > V_P$ (b) $V_R > V_Q > V_P$

   (c) $V_R / V_P = 3$ (d) $V_P / V_Q = \frac{1}{2}$

6. Two bodies, each of mass M, are kept fixed with a separation 2L. A particle of mass m is projected from the midpoint of the line joining their centres, perpendicular to the line. The gravitational constant is G. The correct statement(s) is (are) *(JEE Adv. 2013)*
   (a) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$
   (b) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$
   (c) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$
   (d) The energy of the mass m remains constant

E Subjective Problems

1. Two satellites S_1 and S_2 revolve round a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 hour and 8 hours respectively. The radius of the orbit of S_1 is 10^4 km. When S_2 is closest to S_1, find

(i) the speed of S_2 relative to S_1,
(ii) the angular speed of S_2 as actually observed by an astronaut in S_1. *(1986 - 6 Marks)*

2. Three particles, each of mass m, are situated at the vertices of an equilateral triangle of side length a. The only forces acting on the particles are their mutual gravitational forces. It is desired that each particle moves in a circle while maintaining the original mutual separation a. Find the intial velocity that should be given to each particle and also the time period of the circular motion. *(1988 - 5 Marks)*

3. An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth. *(1990 - 8 Mark)*
(i) Determine the height of the satellite above the earth's surface.
(ii) If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, find the speed with which it hits the surface of the earth.

4. Distance between the centres of two stars is $10a$. The masses of these stars are M and $16M$ and their radii a and $2a$, respectively. A body of mass m is fired straight from the surface of the larger star towards the smaller star. What should be its minimum initial speed to reach the surface of the smaller star? Obtain the expression in terms of G, M and a. *(1996 - 5 Marks)*

5. A body is projected vertically upwards from the bottom of a crater of moon of depth $\frac{R}{100}$ where R is the radius of moon with a velocity equal to the escape velocity on the surface of moon. Calculate maximum height attained by the body from the surface of the moon. *(2003 - 4 Marks)*

H Assertion & Reason Type Questions

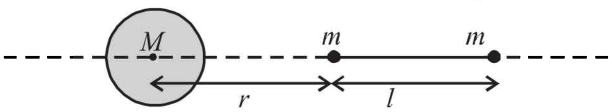
1. **STATEMENT - 1 :** An astronaut in an orbiting space station above the earth experiences weightlessness.
because
STATEMENT - 2 : An object moving the earth under the influence of Earth's gravitational force is in a state of "free-fall". *(2008)*
(a) Statement- 1 is True, Statement-2 is True, Statement-2 is a correct explanation for Statement -1
(b) Statement -1 is True, Statement -2 is True ; Statement-2 is NOT a correct explanation for Statement - 1
(c) Statement - 1 is True, Statement- 2 is False
(d) Statement -1 is False, Statement -2 is True

I Integer Value Correct Type

1. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$. where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be *(2010)*

2. A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is $\frac{1}{4}$th of its value of the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere) *(JEE Adv. 2015)*

3. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length ℓ and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3\ell$ from M, the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$. The value of k is *(JEE Adv. 2015)*



Section-B JEE Main / AIEEE

1. The kinetic energy needed to project a body of mass m from the earth surface (radius R) to infinity is [2002]
(a) $mgR/2$ (b) $2mgR$ (c) mgR (d) $mgR/4$.

2. If suddenly the gravitational force of attraction between Earth and a satellite revolving around it becomes zero, then the satellite will [2002]
(a) continue to move in its orbit with same velocity
(b) move tangentially to the original orbit in the same velocity
(c) become stationary in its orbit
(d) move towards the earth

3. Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is [2002]
(a) $GMm/12R^2$ (b) $GMm/3R^2$
(c) $GMm/8R$ (d) $GMm/6R$.

4. The escape velocity of a body depends upon mass as [2002]
(a) m^0 (b) m^1
(c) m^2 (d) m^3.

5. The time period of a satellite of earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become [2003]
(a) 10 hours (b) 80 hours
(c) 40 hours (d) 20 hours

6. Two spherical bodies of mass M and $5M$ & radii R & $2R$ respectively are released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is [2003]
(a) $2.5R$ (b) $4.5R$ (c) $7.5R$ (d) $1.5R$

7. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45° with the vertical, the escape velocity will be [2003]
(a) $11\sqrt{2}$ km/s (b) 22 km/s
(c) 11 km/s (d) $\frac{11}{\sqrt{2}}$ km/s

8. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is [2004]

(a) $\frac{gR^2}{R+x}$ (b) $\frac{gR}{R-x}$ (c) gx (d) $\left(\frac{gR^2}{R+x}\right)^{1/2}$

9. The time period of an earth satellite in circular orbit is independent of [2004]
(a) both the mass and radius of the orbit
(b) radius of its orbit
(c) the mass of the satellite
(d) neither the mass of the satellite nor the radius of its orbit.

10. If 'g' is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass 'm' raised from the surface of the earth to a height equal to the radius 'R' of the earth is [2004]

(a) $\frac{1}{4}mgR$ (b) $\frac{1}{2}mgR$ (c) $2\,mgR$ (d) mgR

11. Suppose the gravitational force varies inversely as the nth power of distance. Then the time period of a planet in circular orbit of radius 'R' around the sun will be proportional to [2004]

(a) R^n (b) $R^{\left(\frac{n-1}{2}\right)}$ (c) $R^{\left(\frac{n+1}{2}\right)}$ (d) $R^{\left(\frac{n-2}{2}\right)}$

12. The change in the value of 'g' at a height 'h' above the surface of the earth is the same as at a depth 'd' below the surface of earth. When both 'd' and 'h' are much smaller than the radius of earth, then which one of the following is correct ? [2005]

(a) $d=\frac{3h}{2}$ (b) $d=\frac{h}{2}$ (c) $d=h$ (d) $d=2h$

13. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere (you may take $G = 6.67\times 10^{-11}\ Nm^2/kg^2$) [2005]

(a) 3.33×10^{-10} J (b) 13.34×10^{-10} J
(c) 6.67×10^{-10} J (d) 6.67×10^{-9} J

14. Average density of the earth [2005]
(a) is a complex function of g
(b) does not depend on g
(c) is inversely proportional to g
(d) is directly proportional to g

15. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be [2008]
(a) 1.1 km s^{-1} (b) 11 km s^{-1} (c) 110 km s^{-1} (d) 0.11 km s^{-1}

16. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements. [2008]

Statement-1 :
For a mass M kept at the centre of a cube of side 'a', the flux of gravitational field passing through its sides $4\pi GM$.

Statement-2:
If the direction of a field due to a point source is radial and its dependence on the distance 'r' from the source is given as $\frac{1}{r^2}$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.

(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

17. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is : [2009]

(a) $\frac{R}{\sqrt{2}}$ (b) $R/2$ (c) $\sqrt{2}R$ (d) $2R$

18. Two bodies of masses m and $4\,m$ are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is: [2011]

(a) $-\frac{4Gm}{r}$ (b) $-\frac{6Gm}{r}$ (c) $-\frac{9Gm}{r}$ (d) zero

19. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of g and R (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be : [2012]
(a) 6.4×10^{11} Joules (b) 6.4×10^{8} Joules
(c) 6.4×10^{9} Joules (d) 6.4×10^{10} Joules

20. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R? [JEE Main 2013]

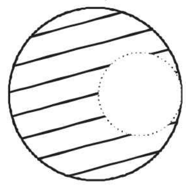
(a) $\frac{5GmM}{6R}$ (b) $\frac{2GmM}{3R}$ (c) $\frac{GmM}{2R}$ (d) $\frac{GmM}{2R}$

21. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is: [JEE Main 2014]

(a) $\sqrt{\frac{GM}{R}}$ (b) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

(c) $\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$ (d) $\frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$

22. From a solid sphere of mass M and radius R, a spherical portion of radius R/2 is removed, as shown in the figure. Taking gravitational potential V = 0 at r = ∞, the potential at the centre of the cavity thus formed is : [JEE Main 2015]
(*G = gravitational constant*)



(a) $\frac{-2GM}{3R}$ (b) $\frac{-2GM}{R}$ (c) $\frac{-GM}{2R}$ (d) $\frac{-GM}{R}$

23. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R; h << R). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere.) [JEE Main 2016]

(a) $\sqrt{gR/2}$ (b) $\sqrt{gR}\left(\sqrt{2}-1\right)$
(c) $\sqrt{2gR}$ (d) $\sqrt{gR}$

7 Gravitation

Section-A : JEE Advanced/ IIT-JEE

A	1.	1.23×10^{-3} rad/s			2.	8.48 hr	3.	$v = \sqrt{\frac{4G}{d}(M_1 + M_2)}$			4.	$h = R$
B	1.	F										
C	1.	(c)	2.	(a)	3.	(b)	4.	(c)	5.	(c)	6.	(d)
	7.	(c)	8.	(a)	9.	(b)	10.	(b)				
D	1.	(b)	2.	(a, c, d)	3.	(a, b)	4.	(a, c)	5.	(b, d)	6.	(b)

E 1. **(i)** $-\pi \times 10^4$ km/hr **(ii)** 3×10^{-4} rad/s 2. $\sqrt{\frac{Gm}{a}}$, $2\pi\sqrt{\frac{a^3}{3Gm}}$

3. **(i)** 6400 km **(ii)** 7.92 km/s 4. $\frac{3}{2}\sqrt{\frac{5GM}{a}}$ 5. $99.5\,R$

H 1. (a)

I 1. 3 2. 2 3. 7

Section-B : JEE Main/ AIEEE

1.	(c)	2.	(b)	3.	(d)	4.	(a)	5.	(c)	6.	(c)
7.	(c)	8.	(d)	9.	(c)	10.	(b)	11.	(c)	12.	(d)
13.	(c)	14.	(d)	15.	(c)	16.	(b)	17.	(d)	18.	(c)
19.	(d)	20.	(a)	21.	(d)	22.	(d)	23.	(b)		

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. We know that $g' = g - R\omega^2 \cos^2 \phi$
At equator, $\phi = 0$, Therefore $g' = g - R\omega^2$

Here $g' = 0$ $\therefore \omega = \sqrt{\frac{g}{R}} = 1.23 \times 10^{-3}$ rad/s

2. **KEY CONCEPT :** According to Kepler's law $T^2 \propto R^3$

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}. \text{ Here } R_1 = R + 6R = 7R$$

and $R_2 = 2.5R + R = 3.5\,R$

$$\Rightarrow \frac{24 \times 24}{T_2^2} = \frac{7 \times 7 \times 7 \times R^3}{3.5 \times 3.5 \times 3.5 \times R^3} \Rightarrow T_2 = 8.48 \text{ hr}$$

3. Increase in P.E. of system

$$= \{(P.E.)_i - (P.E.)_f\}$$

$$= -\left\{\left[-\frac{GM_1M_2}{d} - \frac{GM_1m}{d/2} - \frac{GM_2m}{d/2}\right] - \left[-\frac{GM_1M_2}{d}\right]\right\}$$

$$= \frac{Gm}{d/2}(M_1 + M_2)$$

This increase in P.E. is at the expense of K.E. of mass m

$$\therefore \frac{1}{2}mv^2 = \frac{Gm}{d/2}(M_1 + M_2)$$

where v is the velocity with which mass m is projected.

$$\Rightarrow v = \sqrt{\frac{4G}{d}(M_1 + M_2)}$$

4. $$\frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = -\frac{GMm}{(R+h)} \quad \text{...(ii)}$$

From (i) and (ii)

$$\frac{GMm}{2R} - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

or, $-\frac{1}{2R} = -\frac{1}{R+h} \Rightarrow R + h = 2R$

or, $h = R$

B. True/ False

1. **False**
New Delhi is not on the equatorial plane and geostationary satellite is launched on the equatorial plane.

C. MCQs with ONE Correct Answer

1. **(c)** $g = \frac{GM}{R^2}$ and $g' = \frac{GM}{(0.99R)^2}$

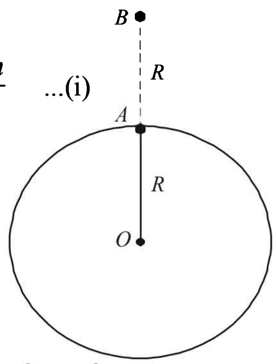
$\therefore \frac{g'}{g} = \left(\frac{R}{0.99R}\right)^2 \Rightarrow g' > g$

2. **(a)** $U_i = -\frac{GMm}{R}$ = Initial potential energy of the system.

$U_f = -\frac{GMm}{2R}$ = Final P.E. of the system.

$\therefore \Delta U = U_f - U_i$

$= -GMm\left[\frac{1}{2R} - \frac{1}{R}\right] = \frac{GMm}{2R}$ ...(i)



But $g = \frac{GM}{R^2}$

$\therefore GM = gR^2$ ... (ii)

From (i) and (ii)

$\Delta U = \frac{gR^2 m}{2R} = \frac{mgR}{2}$

3. **(b)** According to Kepler's law $\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$

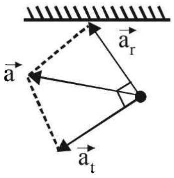
Here $T_1 = 365$ days ; $T_2 = ?$; $R_1 = R$; $R_2 = \frac{R}{2}$

$\Rightarrow T_2 = T_1\left(\frac{R_2}{R_1}\right)^{3/2} = 365\left[\frac{R/2}{R}\right]^{3/2} = 129$ days

4. **(c)** **Note :** A satellite revolving near the earth's surface has a time period of 84.6 min.
We know that as the height increases, the time period increases. Thus the time period of the spy satellite should be slightly greater than 84.6 minutes.

$\therefore T_s = 2$ hr

5. **(c)** The components of acceleration are as shown

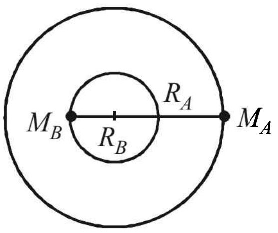


$a = \vec{a}_r + \vec{a}_t$

The resultant of transverse and radial component of the acceleration is represented by $\vec{a}$

6. **(d)** **Note :** The gravitational force of attraction between the stars will provide the necessary centripetal forces. In this case angular velocity of both stars is the same.

Therefore time period remains the same. $\left(\omega = \frac{2\pi}{T}\right)$.



7. **(c)** **For $r \geq R$**

Force on the test mass m is $F = m \times |E_g|$

Where E_g is the gravitational field intensity at the point of observation

$\therefore \frac{mv^2}{r} = m \times \left[\frac{GM}{r^2}\right]$ where M is the total mass of the spherical system.

$\therefore v \propto \frac{1}{\sqrt{r}}$

For r < R Again $F' = m|E_g'|$

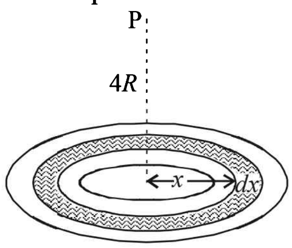
$\therefore \frac{mv^2}{r} = m\left[\frac{GM}{R^3} \times r\right]$

$\Rightarrow v \propto r$

8. **(a)** Let us consider a circular elemental area of radius x and thickness dx. The area of the shaded portion $= 2\pi x dx$.
Let dm be the mass of the shaded portion.

$\therefore \frac{\text{Mass}}{\text{area}} = \frac{M}{\pi(4R^2) - \pi(3R)^2}$

$= \frac{dm}{2\pi x dx}$



$\therefore dm = \frac{2M}{7R^2} x dx$

The gravitational potential of the mass dm at P is

$dV = \frac{-G\,dm}{\sqrt{(4R)^2 + x^2}} = -\frac{G}{\sqrt{16R^2 + x^2}} \times \frac{2M}{7R^2} x dx$

$= \frac{-2GM}{7R^2} \frac{x dx}{\sqrt{16R^2 + x^2}}$ (1)

Suppose $16R^2 + x^2 = t^2$

$\Rightarrow 2x dx = 2t dt \Rightarrow x dx = t dt$

Also for $x = 3R$, $t = 5R$

and for $x = 4R$, $t = 4\sqrt{2}R$

On integrating equation (1), taking the above limits, we get

$V = -\int_{5R}^{4\sqrt{2}R} \frac{2GM}{7R^2} dt = \frac{-2GM}{7R^2}[t]_{5R}^{4\sqrt{2}R}$

$= \frac{-2GM}{7R^2}\left[4\sqrt{2}R - 5R\right] \Rightarrow V = \frac{-2GM}{7R}(4\sqrt{2} - 5)$

Now $\frac{W_{P\infty}}{1} = V_\infty - V_P = -V_P$ $[\because V_\infty = 0]$

$\therefore W_{P\infty} = \frac{2GM}{7R}(4\sqrt{2} - 5)$

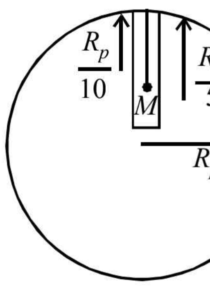
9. **(b)** V is the orbital velocity. If V_C is the escape velocity then $V_e = \sqrt{2}V$. The kinetic energy at the time of ejection

$KE = \frac{1}{2}mV_e^2 = \frac{1}{2}m(\sqrt{2}V)^2 = mV^2$

10. **(b)** $R_p = \frac{R_e}{10} = 6 \times 10^5 m$

The mass of the wire $= 10^{-3} \times 1.2 \times 10^5 = 120$ kg

Let g_{pM} be the acceleration due to gravity at point M which is the mid point of the wire and is at a depth of $\frac{R_p}{10}$.



Let g_p be the acceleration due to gravity at the surface of the planet.

$$g_p = \frac{4}{3}\pi\rho G R_P \; ; \; g_e = \frac{4}{3}\pi\rho G R_E$$

$$\therefore \quad \frac{g_p}{g_e} = \frac{R_p}{R_E} = \frac{1}{10}$$

$$\therefore \quad g_p = \frac{10}{10} = 1 \text{ ms}^{-2}$$

and $g_{pM} = g_p\left[1 - \frac{R_p/10}{R_p}\right] = 1[1-0.1] = 0.9 \text{ ms}^{-2}$

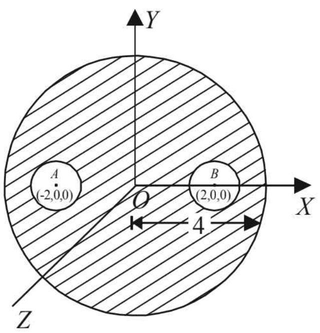
$\therefore$ Force = mass of wire × g_{pM} = 120 × 0.9 = 108 N

D. MCQs with ONE or MORE THAN ONE Correct

1. (b) **KEY CONCEPT :** The centripetal force is provided by the gravitational force of attraction $mR\omega^2 = GMmR^{-5/2}$

$$\Rightarrow \frac{mR \times 4\pi^2}{T^2} = \frac{GMm}{R^{5/2}} \Rightarrow T^2 \propto R^{7/2}$$

2. (a,c,d) The gravitational field intensity at the point O is zero (as the cavities are symmetrical with respect to O). Now the force acting on a test mass m_0 placed at O is given by
$F = m_0 E = m_0 \times 0 = 0$
Now, $y^2 + z^2 = 36$ represents the equation of a circle with centre (0, 0, 0) and radius 6 units the plane of the circle is perpendicular to x-axis.



Note : Since the spherical mass distribution behaves as if the whole mass is at its centre (for a point outside on the sphere) and since all the points on the circle is equidistant from the centre of the sphere, the circle is a gravitational equipotential.
The same logic holds good for option (d).

3. (a,b) For $r > R$, the gravitational field is $F = \frac{GM}{r^2}$

$$\therefore \quad F_1 = \frac{GM}{r_1^2} \text{ and } F_2 = \frac{GM}{r_2^2} \Rightarrow \frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$$

For $r < R$, the gravitational field is $F = \frac{GM}{R^3} \times r$

$$\therefore \quad F_1 = \frac{GM}{R^3} \times r_1 \text{ and } F_2 = \frac{GM}{R^3} \times r_2$$

$$\Rightarrow \quad \frac{F_1}{F_2} = \frac{r_1}{r_2}$$

4. (a, c) Force on satellite is always towards earth, therefore, acceleration of satellite S is always directed towards centre of the earth. Net torque of this gravitational force F about centre of earth is zero. Therefore, angular momentum (both in magnitude and direction) of S about centre of earth is constant throughout. Since the force F is conservative in nature, therefore mechanical energy of satellite remains constant. Speed of S is maximum when it is nearest to earth and minimum when it is farthest.

5. (b, d) Let the mass of P be m.

$$\text{Then } m = \rho \times \frac{4}{3}\pi r^3 = \rho \times \frac{4}{3}\pi\left[\frac{A}{4\pi}\right]^{3/2}$$

$$\text{The mass of } Q = \rho \times \frac{4}{3}\pi\left[\frac{4A}{4\pi}\right]^{3/2} = 8\text{ m}$$

$\therefore$ The mass of R = 9 m
If the radius of $P = r$
Then the radius of $Q = 2r$

$$\left[\because r_Q = \left(\frac{4A}{4\pi}\right)^{3/2} = 2\left(\frac{A}{4\pi}\right)^{3/2}\right]$$

and radius of $R = 9^{1/3} r$

$$\left[\begin{array}{l}\because M_R = M_P + M_Q \\ r_R^3 = r^3 + (2r)^3 = 9r^3\end{array}\right]$$

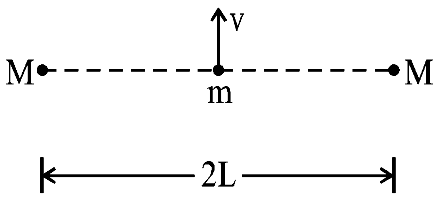
$$\text{Now, } v_P = \sqrt{\frac{2GM_P}{R_p}} = \sqrt{\frac{2Gm}{r}}$$

$$v_Q = \sqrt{\frac{2GM_Q}{R_Q}} = \sqrt{\frac{2G\,(8\text{m})}{2r}} = 2v_P$$

$$v_R = \sqrt{\frac{2G\,(9\text{ m})}{9^{1/3} r}} = 9^{1/3} v_P$$

6. (b) $\frac{1}{2}mv^2 = 2\left[\frac{\text{GMm}}{\text{L}}\right] \Rightarrow \text{v} = 2\sqrt{\frac{\text{GM}}{\text{L}}}$

The potential energy is a combined property of the three mass system. The kinetic energy of mass m is only its energy which decreases as it moves.
(b) is the correct option.



E. Subjective Problems

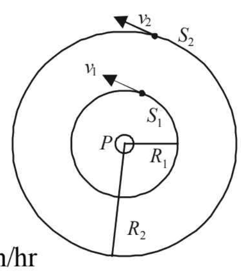
1. (i) According to Kepler's third law

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \Rightarrow R_2^3 = R_1^3 \times \frac{T_2^2}{T_1^2}$$

$$\therefore \quad R_2^3 = \left(10^4\right)^3 \times \frac{8^2}{1^2} = 64 \times 10^{12}$$

$\Rightarrow$ $R_2 = 4 \times 10^4$ km.
Linear speed of satellite S_1

$$v_1 = \frac{2\pi R_1}{T_1} = \frac{2\pi \times 10^4}{1} = 2\pi \times 10^4 \text{ km/hr}$$



Linear speed of satellite S_2,

$$v_2 = \frac{2\pi R_2}{T_2} = \frac{(2\pi)(4\times10^4)}{8} = \pi\times10^4 \text{ km/hr}$$

The speed of satellite S_2 w.r.t. S_1

$$= v_2 - v_1 = \pi\times10^4 - 2\pi\times10^4 = -\pi\times10^4 \text{ km/hr}$$

(ii) Angular speed of S_2 w.r.t. S_1

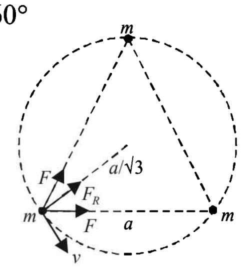
$$= \frac{v_r}{R_r} = \frac{v_2 - v_1}{R_2 - R_1} = \frac{3.14\times10^4\times5/18}{3\times10^4\times10^3} = 3\times10^{-4} \text{ rad/s}$$

2. The radius of the circle $r = \frac{2}{3}\sqrt{a^2 - \frac{a^2}{4}} = \frac{a}{\sqrt{3}}$

Let v be the velocity given. The centripetal force is provided by the resultant gravitational attraction of the two masses.

$$F_R = \sqrt{F^2 + F^2 + 2F^2\cos60°}$$

$$= \sqrt{3}F = \sqrt{3}G\frac{m\times m}{a^2}$$



$$\therefore \sqrt{3}G\frac{m^2}{a^2} = \frac{mv^2}{r}$$

$$\left(\frac{mv^2}{r} = \text{centripetal force}\right)$$

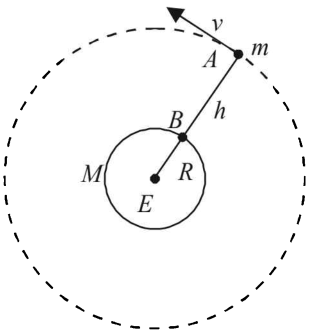
$$v^2 = \frac{\sqrt{3}Gmr}{a^2} = \frac{\sqrt{3}Gma}{a^2\times\sqrt{3}} \Rightarrow v = \sqrt{\frac{Gm}{a}}$$

Time period of circular motion

$$T = \frac{2\pi r}{v} = \frac{2\pi a/\sqrt{3}}{\sqrt{\frac{Gm}{a}}} = 2\pi\sqrt{\frac{a^3}{3Gm}}$$

3. (i) **KEY CONCEPT :** Since the satellite is revolving in a circular orbit, the centripetal force is provided by the gravitational pull.

$$\frac{mv^2}{(R+h)} = \frac{GMm}{(R+h)^2}$$



$$\therefore v^2 = \frac{GM}{R+h}$$

But $v = \frac{1}{2}v_e = \frac{1}{2}\sqrt{\frac{2GM}{R}}$

$$\therefore \frac{1}{4}\left(\frac{2GM}{R}\right) = \frac{GM}{R+h}$$

$\Rightarrow 2R + 2h = 4R \Rightarrow h = R = 6400$ km.

(ii) **KEY CONCEPT :** When the satellite is stopped, its kinetic energy is zero. When it falls freely on the Earth, its potential energy decreases and converts into kinetic energy.

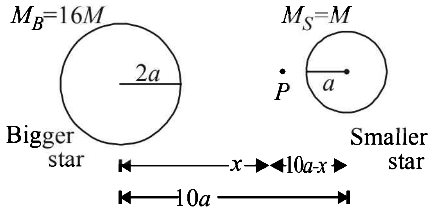
$\therefore$ (P.E.)$_A$ − (P.E.)$_B$ = K.E.

$$\Rightarrow \frac{-GMm}{2R} - \left(\frac{-GMm}{R}\right) = \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{\frac{GM}{R}} = \sqrt{gR} = \sqrt{9.8\times6.4\times10^6}$$

= 7920 m/s = 7.92 km/s

4.



The force of attraction is zero at say x from the bigger star. Then force on mass m due to bigger star = Force on mass m due to small star

$$\frac{GM_Bm}{x^2} = \frac{GM_Sm}{(10a-x)^2} \Rightarrow \frac{16M}{x^2} = \frac{M}{(10a-x)^2} \Rightarrow x = 8a$$

If we throw a mass m from bigger star giving it such a velocity that is sufficient to bring it to P, then later on due to greater force by the star M_S it will pull it towards itself [without any external energy thereafter].

The energy of the system (of these masses) initially = Final energy when m is at P

$$-\frac{GM_BM_S}{10a} - \frac{GM_Bm}{2a} - \frac{GM_Sm}{8a} + \frac{1}{2}mv^2$$

$$= -\frac{GM_BM_S}{10a} - \frac{GM_Bm}{8a} - \frac{GM_Sm}{2a}$$

$[\because M_B = 16M;\ M_S = M]$

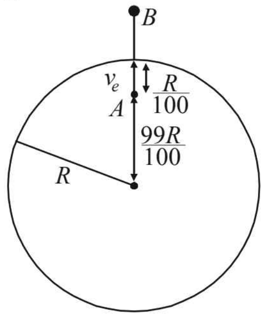
$$\therefore v = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

5. Total energy at A = Total energy at B

(K.E.)$_A$ + (P.E.)$_A$ = (P.E.)$_B$

$$\Rightarrow \frac{1}{2}m\times\frac{2GM}{R} + \left[\frac{-GMm}{2R^3}\left\{3R^2 - \left(\frac{99R}{100}\right)^2\right\}\right] = -\frac{GMm}{R+h}$$

On solving we get $h = 99.5\,R$.



H. Assertion & Reason Type Questions

1. **(a)** The normal force exerted by the astronaut on orbiting space station is zero (until the astronaut exerts some muscular force). Therefore the apparent weight of astronaut in an orbiting space station is zero. Astronaut is called in a state of weightlessness. This is because astronaut as well as space-ship are freely falling bodies. Statement - 1 is true, statement - 2 is true and statement - 2 is the correct explanation of statement - 1.

I. Integer Value Correct Type

1. We know that $v = \sqrt{2gR}$

$$\therefore \frac{v_p}{v} = \sqrt{\frac{g_p}{g}\times\frac{R_p}{R}} \quad ...(i)$$

Given $\frac{g_p}{g_e} = \frac{\sqrt{6}}{11}$ ...(ii)

Also $g = \frac{4}{3}\pi G\rho R \quad \therefore \frac{g_p}{g} = \frac{\rho_p}{\rho} \times \frac{R_p}{R}$

$\therefore \frac{\sqrt{6}}{11} = \frac{2}{3} \times \frac{R_p}{R} \quad \left[\because \frac{\rho_p}{\rho} = \frac{2}{3}(\text{given})\right]$

$\therefore \frac{R_p}{R} = \frac{3\sqrt{6}}{22}$ ...(iii)

From (i), (ii) & (iii) $\frac{v_p}{v} = \sqrt{\frac{\sqrt{6}}{11} \times \frac{3\sqrt{6}}{22}} = \sqrt{\frac{3\times 6}{11\times 22}} = \frac{3}{11}$

$\therefore v_p = \frac{3}{11} \times v = \frac{3}{11} \times 11\,\text{km/s} = \mathbf{3}\,\text{km/s}$

2. (2) Let h be the height to which the bullet rises

then, $g^1 = g\left(1+\frac{h}{R}\right)^{-2}$

$\Rightarrow \frac{g}{4} = g\left(1+\frac{h}{R}\right)^{-2}$

$\Rightarrow h = R$

We know that $v_e = \sqrt{\frac{2GM}{R}} = v\sqrt{N}$ (given) ...(i)

Now applying conservation of energy for the throw

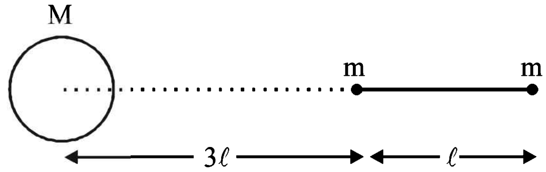
Loss of kinetic energy = Gain in gravitaional potential energy

$\therefore \frac{1}{2}mv^2 = -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right)$

$\therefore v = \sqrt{\frac{GM}{R}}$ ...(ii)

Comparing (i) & (ii) $\quad N = 2$

3. (7) For the tension in the rod to be zero, the force on both the masses m and m should be equal in magnitude and direction. Therefore



$\frac{GMm}{(4\ell)^2} + \frac{Gmm}{\ell^2} = \frac{GMm}{(3\ell)^2} - \frac{Gmm}{\ell^2}$

$\therefore 2m = M\left[\frac{1}{9} - \frac{1}{16}\right]$

$\therefore m = \frac{7M}{288}$

$K = 7$

Section-B JEE Main/ AIEEE

1. (c) $K.E = \frac{1}{2}mv_e^2$ where v_e = escape velocity = $\sqrt{2gR}$

$\therefore K.E = \frac{1}{2}m \times 2gR = mgR$

2. (b) Due to inertia of motion it will move tangentially to the original orbit in the same velocity.

3. (d) Energy required = (Potential energy of the Earth -mass system when mass is at distance $3R$) – (Potential energy of the Earth -mass system when mass is at distance $2R$)

$= \frac{-GMm}{3R} - \left(\frac{-GMm}{2R}\right) = \frac{-GMm}{3R} + \frac{GMm}{2R}$

$= \frac{-2GMm + 3GMm}{6R} = \frac{GMm}{6R}$

4. (a) Escape velocity, $v_e = \sqrt{2gR} = \sqrt{\frac{2GM}{R}} \Rightarrow V_e \propto m^0$

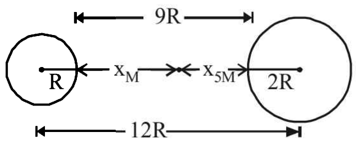
Where M, R are the mass and radius of the planet respectively. In this expression the mass of the body (m) is not present showing that the escape velocity is independent of the mass.

5. (c) According to Kepler's law of planetary motion $T^2 \propto R^3$

$\therefore T_2 = T_1\left(\frac{R_2}{R_1}\right)^{3/2} = 5 \times \left[\frac{4R}{R}\right]^{3/2} = 5 \times 2^3 = 40$ hour

6. (c) The gravitational force acting on both the masses is the same. We know that

Force = mass × acceleration.



For same force, acceleration $\propto \frac{1}{\text{mass}}$

$\therefore \frac{a_{5M}}{a_M} = \frac{M}{5M} = \frac{1}{5}$ .....(i)

Let t be the time taken for the two masses to collide and x_{5M}, x_M be the distance travelled by the mass $5M$ and M respectively.

For mass 5*M*

$u = 0,\ S = x_{5M},\ t = t,\ a = a_{5M}$

$S = ut + \frac{1}{2}at^2 \quad \therefore x_{5M} = \frac{1}{2}a_{5M}t^2$ ....(ii)

For mass *M*

$u = 0, s = x_M, t = t,\ a = a_M$

$\therefore s = ut + \frac{1}{2}at^2 \Rightarrow x_M = \frac{1}{2}a_M t^2$ ... (iii)

Dividing (ii) by (iii)

$\frac{x_{5M}}{x_M} = \frac{\frac{1}{2}a_{5M}t^2}{\frac{1}{2}a_M t^2} = \frac{a_{5M}}{a_M} = \frac{1}{5}$ [From (i)]

$\therefore 5x_{5M} = x_M$ ....(iv)

From the figure it is clear that

$x_{5M} + x_M = 9R$ ....(v)

Where O is the point where the two spheres collide.

From (iv) and (v)

$$\frac{x_M}{5} + x_M = 9R$$

$$\therefore\ 6x_M = 45R \qquad \therefore\ x_M = \frac{45}{6}R = 7.5R$$

7. **(c)** $v_e = \sqrt{2gR}$

The escape velocity is independent of the angle at which the body is projected.

8. **(d)** Gravitational force provides the necessary centripetal force.

$$\therefore \frac{mv^2}{(R+x)} = \frac{GmM}{(R+x)^2} \text{ also } g = \frac{GM}{R^2}$$

$$\therefore v^2 = \frac{gR^2}{R+x} \Rightarrow v = \left(\frac{gR^2}{R+x}\right)^{1/2}$$

9. **(c)** We have, $\frac{mv^2}{R+x} = \frac{GmM}{(R+x)^2}$

x = height of satellite from earth surface

m = mass of satellite

$$\Rightarrow v^2 = \frac{GM}{(R+x)} \text{ or } v = \sqrt{\frac{GM}{R+x}}$$

$$T = \frac{2\pi(R+x)}{v} = \frac{2\pi(R+x)}{\sqrt{\frac{GM}{R+x}}}$$

which is independent of mass of satellite

10. **(b)** $\therefore \Delta U = \frac{-GmM}{2R} + \frac{GmM}{R};\ \Delta U = \frac{GmM}{2R}$

Now $\frac{GM}{R^2} = g;\ \therefore \frac{GM}{R} = gR\ \therefore \Delta U = \frac{1}{2}mgR$

11. **(c)** $F = KR^{-n} = MR\omega^2 \Rightarrow \omega^2 = KR^{-(n+1)}$

or $\omega = KR^{\frac{-(n+1)}{2}}$

$$\frac{2\pi}{T} \propto R^{\frac{-(n+1)}{2}} \qquad \therefore T \propto R^{\frac{+(n+1)}{2}}$$

12. **(d)** Variation of g with altitude is, $g_h = g\left[1 - \frac{2h}{R}\right]$;

variation of g with depth is, $g_d = g\left[1 - \frac{d}{R}\right]$

Equating g_h and g_d, we get $d = 2h$

13. **(c)** Workdone, $W = \Delta U = U_f - U_i = 0 - \left[\frac{-GMm}{R}\right]$

$$W = \frac{6.67\times10^{-11}\times100}{0.1}\times\frac{10}{1000} = 6.67\times10^{-10}\text{ J}$$

14. **(d)** $g = \frac{GM}{R^2} = \frac{G\rho\times V}{R^2} \Rightarrow g = \frac{G\times\rho\times\frac{4}{3}\pi R^3}{R^2}$

$g = \frac{4}{3}\rho\pi G.R$ where $\rho \to$ average density

15. **(c)** $$\frac{(v_e)_p}{(v_e)_e} = \frac{\sqrt{\frac{2GM_p}{R_p}}}{\sqrt{\frac{2GM_e}{R_e}}} = \sqrt{\frac{M_p}{M_e}\times\frac{R_e}{R_p}}$$

$$= \sqrt{\frac{10M_e}{M_e}\times\frac{R_e}{R_e/10}} = 10$$

$$\therefore (v_e)_p = 10\times(v_e)_e = 10\times11 = 110\ km/s$$

16. **(b)** Gravitational flux through a closed surface is given by

$\int E_g\, dS = -4\pi GM$

where, M = mass enclosed in the closed surface

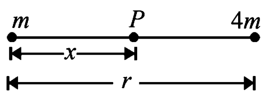
This relationship is valid when $|E_g| \propto \frac{1}{r^2}$.

17. **(d)** We know that $\frac{g'}{g} = \frac{R^2}{(R+h)^2}$

$$\therefore \frac{g/9}{g} = \left[\frac{R}{R+h}\right]^2 \qquad \therefore h = 2R$$

18. **(c)** Let the gravitational field at P, distant x from mass m, be zero.

$$\therefore \frac{Gm}{x^2} = \frac{4Gm}{(r-x)^2} \qquad \Rightarrow x = \frac{r}{3}$$



Gravitational potential at P, $V = -\frac{Gm}{\frac{r}{3}} - \frac{4Gm}{\frac{2r}{3}} = -\frac{9Gm}{r}$

19. **(d)** The required energy for this work is given by

$$\frac{GMm}{R} = mgR$$

$= 1000\times10\times6400\times10^3$

$= 6.4\times10^{10}$ Joules

20. **(a)** Applying energy conservation

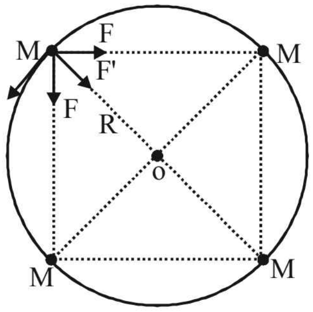
$$k + \left(-\frac{GMm}{R}\right) = \frac{1}{2}mv_0^2 + \left(-\frac{GMm}{R+h}\right)$$

$$k - \frac{GMm}{R} = \frac{1}{2}m\left(\frac{Gm}{R+2R}\right) - \frac{GMm}{R+2R}$$

$$\therefore k = \frac{5GMm}{6R}$$

21. **(d)** $2F\cos45° + F' = \frac{Mv^2}{R}$ (From figure)

Where $F = \frac{GM^2}{(\sqrt{2}R)^2}$ and $F' = \frac{GM^2}{4R^2}$



$$\Rightarrow \frac{2\times GM^2}{\sqrt{2}(R\sqrt{2})^2}+\frac{GM^2}{4R^2}=\frac{Mv^2}{R}$$

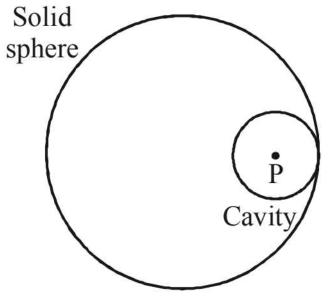
$$\Rightarrow \frac{GM^2}{R}\left[\frac{1}{4}+\frac{1}{\sqrt{2}}\right]=Mv^2$$

$$\therefore \quad v=\sqrt{\frac{Gm}{R}\left(\frac{\sqrt{2}+4}{4\sqrt{2}}\right)}=\frac{1}{2}\sqrt{\frac{Gm}{R}(1+2\sqrt{2})}$$

22. **(d)** Due to complete solid sphere, potential at point P

$$V_{sphere}=\frac{-GM}{2R^3}\left[3R^2-\left(\frac{R}{2}\right)^2\right]$$

$$=\frac{-GM}{2R^3}\left(\frac{11R^2}{4}\right)=-11\frac{GM}{8R}$$



Due to cavity part potential at point P

$$V_{cavity}=-\frac{3}{2}\frac{\frac{GM}{8}}{\frac{R}{2}}=-\frac{3GM}{8R}$$

So potential at the centre of cavity

$$=V_{sphere}-V_{cavity}=-\frac{11GM}{8R}-\left(-\frac{3}{8}\frac{GM}{R}\right)=\frac{-GM}{R}$$

23. **(b)** For h << R, the orbital velocity is $\sqrt{gR}$

Escape velocity = $\sqrt{2gR}$

$\therefore$ The minimum increase in its orbital velocity

$=\sqrt{2gR}-\sqrt{gR}=\sqrt{gR}\,(\sqrt{2}-1)$